Scuola Matematica Interuniversitaria

Summer Course, Perugia 2017

Introduction to Partial Differential Equations

Fausto Ferrari, Raffaella Servadei

The course will provide an introduction to partial differential equations. A basic knowledge of real analysis (Lebesgue integration, L^p spaces, Fourier transform) is required. In the second part of the course some further key tools in the theory of elliptic PDEs will be introduced and some basic notions of functional analysis will be briefly recalled. The lessons will be calibrated in relation to the audience.

Syllabus

1. The single first order equation
    - Introduction to the characteristic method
    - The Cauchy problem
    - Second order equations: classification of quasi-linear second order equations
2. The diffusion equation
    - Uniqueness
    - The fundamental solution
    - Symmetric random walk
    - Diffusion Drift Reaction
    - Some applications
3. The Laplace equation
    - Harmonic functions
    - Fundamental solution
    - Green function
    - Maximum principles and Harnack inequality
    - Variational solutions versus Viscosity solutions

4. Wave equation

   - Separation of variables
   - The d'Alembert formula
   - The fundamental solution
   - Non-homogeneous equation. Duhamel's method

5. Nonlocal operators:

   - The fractional Laplace operator and the fractional derivative
   - Riemann-Liouville derivative
   - Grünwald-Letnikov derivative, Marchaud derivative.

6. A brief introduction to Sobolev Spaces

   - Weak derivatives and elementary properties
   - Sobolev inequalities

7. Variational formulation of elliptic problems

   - The Poisson problem: weak and variational formulation
   - General elliptic equations in divergence form
   - Lax-Milgram Theorem
   - Regularity of weak solutions
   - Nonlocal problems of fractional Laplacian type

References

- F. John, *Partial Differential Equations*, Applied Mathematical Sciences 1, Springer.
- S. Salsa, *Partial differential equations in action. From modelling to theory*, Chapters 7-9, Second edition, Unitext, 86, Springer, 2015.
- L.C. Evans, *Partial differential equations. Part II*

  Second edition, Graduate Studies in Mathematics, 19, American Mathematical Society, 2010.
- H. Brezis, *Functional analysis, Sobolev spaces and partial differential equations*, Chapters 7-10, Universitext, Springer, New York, 2011.