***Orientation Score Theory and its Solutions to Cortical PDE, ODE and Wavelet Models.***

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**Lecture 1** (invertible multi-orientation image representations)
*Introduction to Image Processing via Invertible Orientation Scores.*
**Content:** In this 1st lecture we aim for introduction and motivation of the orientation score framework:
- What is an invertible orientation score?
- Why do we need invertible orientation scores?
- Why must we apply left-invariant processing on them?
- What medical image analysis applications can we tackle with them?

**Exercise Topics**:
- Condition numbers of the transformation.
- Proofs of general unitarity results.
- the Lie group structure on the domain of an orientation score.

**Lecture 2** (design of multi-orientation image representations)
*Design of Invertible Orientation Scores of 2D and 3D images.*
**Content:** In this 2nd lecture we focus on the following topics:
- Design of 2D Cake-wavelets.
- Design of 3D Cake-wavelets.
- Relating the forward orientation score transform to inverse Fourier transform on SE(2).
- References to Tutorials of the Lie Analysis Mathematica 11 Package.

**Exercise Topics:**
- Design of cake-wavelets in Fourier domain (B-splines, Spherical Harmonics, Func-Transform)
- Analytic versions of cake-wavelets in both spatial and Fourier domains. (modified Zernike basis)
- Questions on playing with cake-wavelet design in our Lie Analysis package.

**Lecture 3** (solutions to cortical models for contour completion and enhancement)
*Exact Solutions to Linear Left-Invariant PDE's on SE(2) and on SE(3).*
**Content:** In this 3rd lecture we focus on the following topics:
- Exact solutions to Mumford's direction Process on SE(2).
- Exact solutions to Hypo-elliptic Diffusion on SE(2).
- Exact solutions to Mumford's direction Process on SE(3).

**Exercise Topics:**
-A review on well-known (and related) exact solutions for diffusions and Tikhonov regularization on a sphere.
-Method of separation exercises tackling specific sub-problems on SE(d).

**Lecture 4** (crossing-preserving diffusion via `straight curves' in orientation scores)
*Crossing-preserving diffusions via locally adaptive frames in invertible orientation scores.*
**Content:** In this 4th lecture we focus on the following topics:
- Exponential Curve fits of order 1.
- Exponential Curve fits of order 2.
- Crossing-Preserving Diffusions on SE(2).
- Crossing-Preserving Diffusions on SE(3).

**Exercise Topics:**

- Compute exponential curves in SE(2).
- Euler-Lagrange equations of exponential curve fits of order 1 on SE(2).
- Euler-Lagrange equations of exponential curve fits of order 2 on SE(2).
- The minus Cartan connection: "Straight curves" vs. "shortest curves".

**Lecture 5** (tracking via `shortest curves' in orientation scores)
*Globally Optimal Sub-Riemannian geodesics in the projective line bundle.*
*Improvements via Asymmetric Finsler geometry: key-points instead of cusps !*
**Content:** In this 5th lecture we illustrate geodesic tracking in orientation scores:
- Sub-Riemannian Geodesics in SE(2) and in the projective line bundle, and vessel-tracking applications.
- Sub-Riemannian Geodesics in SE(3) and DW-MRI applications.
- The problem of cusps in spatial projections of SR geodesics.
- The solution to the problem of cusps: Turn off the reverse gear of the Reeds-Shepp car.
**Exercise Topics:**
- Derive the Hamiltonian equations for sub-Riemannian geodesics in SE(2).
- Preservation laws and symmetries of sub-Riemannian geodesics in SE(d), d=2,3.
- Analyze the shape of the sub-Riemannian spheres on SE(2) and their intersection with the 1st Maxwell set.
- Analyze the shape of the sub-Riemannian spheres on the projective line bundle and their intersection with the 1st Maxwell set.
- Apply a vessel-tracking in a challenging industrial medical imaging example in a prepared Mathematica 11 notebook**.**