Harmonic Analysis for Modeling Visual Cortical Functions

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Lecture 1 Mathematical description of neural behaviors
1.1 Receptive fields
1.1.1 The Linear-Nonlinear-Poisson model
1.1.2 Reverse correlation and experimental receptive field morphologies
1.1.3 Spatial and spatio-temporal Gabor models for V1
1.2 Connectivities
1.2.1 Short range nonlinearities: suppression and normalization
1.2.2 Connectivity kernels and the neural field equation
1.2.3 Simple single and multi layer neural network models_EXERCISES
1.3 Synaptic plasticity
1.3.1 Basic Hebbian learning
1.3.2 Normalization, convergence and modified learning rules
1.3.3 Principal Component Analysis_EXERCISES

Lecture 2 Group theory: symmetry features in data spaces
2.1 Locally compact groups
2.1.1 Group representations and Fourier transform_EXERCISES
2.1.2 Coherent states and square integrability
2.1.4 Semidirect products with a normal abelian subgroup
2.2 Lie groups
2.2.1 Lie algebras and their representations
2.2.2 The Heisenberg group, Gabor systems and musical scores
2.2.3 The Euclidean motion groups_EXERCISES

Lecture 3 Systems of linear analyzers
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3.1.1 Positive de_nite kernels and Mercer's theorem
3.1.2 Rigidity of RKHS: the pointwise evaluation
3.1.3 Convolution algebras and the group RKHS
3.1.4 An example: the Bargmann-Fock space _ EXERCISES
3.2 Frames and bases from families of vectors
3.2.1 Analysis, synthesis, Gram and frame operators
3.2.2 Basic characterizations
3.2.3 Spaces invariant under translations _ EXERCISES
3.2.4 Sampling theorems

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4.1.1 Independent Component Analysis
4.1.2 Sparsity and the Olshausen-Field result _ EXERCISES
4.1.3 More on L1 minimization: compressed sensing
4.2 Classi_cation and clustering
4.2.1 Kernel PCA
4.2.2 Graph Laplacians _ EXERCISES
4.2.3 Sketches of deep learning

Lecture 5 Spatio-temporal processing in V1
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5.1.1 The quasi-Galilean structure
5.1.2 Fokker Planck equations
5.1.3 Experimental evidence of preactivation
5.2 Objects and motion detection
5.2.1 Orientations for spatial clustering
5.2.1 Direct graphs and diffusion
5.2.2 Psychophysical experiments