Scuola Matematica Interuniversitaria
Summer Course, Perugia, 2016

Introduction to Partial Differential Equations

Ugo Gianazza, Giuseppe Savaré
Università di Pavia

The course will provide an introduction to the classical functional analytic approach to linear second-order partial differential equations, based on Sobolev spaces and variational formulations. A basic knowledge of real analysis (Lebesgue integration, L^p spaces, absolutely continuous functions in one dimension) is required. A few basic notions of functional analysis will be briefly recalled during the course.

Syllabus

A brief introduction to Sobolev spaces

- Weak derivatives, elementary properties, calculus rules
- Approximation, extensions and traces
- Sobolev and Poincaré inequalities, compactness.

Second-order linear elliptic equations

- A brief review on classical properties of Harmonic functions and Laplace equation.
- Weak solutions, variational formulation
- Lax-Milgram theorem and existence of weak solutions, Fredholm alternative.
- Regularity
- Maximum principles and Harnack's inequality
- Eigenvalues and eigenfunctions

Second-order linear parabolic equations

- Heat equation and fundamental solution
- Spaces of time-dependent functions.
- Weak solutions, variational formulation, energy estimates
- Existence and uniqueness: J.L. Lions' Theorem.
- Regularity
- Maximum principles and Harnack inequalities
- The semigroup approach: Hille-Yosida theorem.

Second-order linear hyperbolic equations

- Solutions to the Wave equation by spherical means
- Weak solutions, energy estimates
- Existence and uniqueness
- Regularity
- Finite speed propagation

References

L.C. Evans. *Partial differential equations.* Part II
Second edition. Graduate Studies in Mathematics, 19. American Mathematical Society, 2010.

S. Salsa *Partial differential equations in action. From modelling to theory.* Chapters 7–9
Second edition. Unitext, 86. Springer, 2015.

H. Brezis *Functional analysis, Sobolev spaces and partial differential equations.* Chapters 7–10
Universitext. Springer, New York, 2011.