DIFFERENTIAL GEOMETRY

SMI, Perugia 2016

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Prerequisites

Linear algebra: vector spaces, scalar products, norms, symmetric endomorphisms, eigenvalues, multilinear forms.
Differential and integral calculus of one and several real variables.
Elementary theory of ODE's.

Syllabus

1. **Differentiable manifolds.**
1.1 Definitions, examples.
1.2 Tangent space, differentiable maps.
1.3 Immersions, submersions, submanifolds. Regular value theorem.
1.4 Lie groups.
1.5 Vector fields, bracket. Flows. Lie derivative.
1.6 Distributions. Frobenius theorem.

2. **Tensors and differential forms.**
2.1 Tensor algebra, exterior algebra.
2.2 Tensor bundles. Sections.
2.3 Exterior differential. Lie derivative of forms.
2.4 De Rham cohomology. Poincaré lemma.
2.5 Integration on manifolds.

3. **Vector bundles**
3.1 Definitions. Constructions.
3.2 Connections in vector bundles. Parallel transport.

4. **Riemannian manifolds**
3.1 Definitions. Examples. Riemannian coverings.
3.2 Levi Civita connection.
3.3 Riemannian curvature. Sectional curvature.
3.4 Geodesics.

Recommended texts:

[1] F. Warner, Foundations of differentiable manifolds and Lie groups. Corrected reprint of the 1971 edition. Graduate Texts in Mathematics, 94. *Springer-Verlag, New York-Berlin,* 1983. ix+272 pp.

[2] W. Boothby, An introduction to differentiable manifolds and Riemannian geometry. Second edition. Pure and Applied Mathematics, 120. Academic Press, Inc., Orlando, FL, 1986. xvi+430 pp.

NB 1) The prerequisites (especially ODE's) should be regarded as mere wishful thinking.
2) The syllabus will be adapted according to the participants' background.
3) I can lecture in English or Italian, according to the participants' preferences.