Semi-simple Lie Algebras

Lie Algebras play an important role in many areas of mathematics: Algebra, differential and algebraic geometry, mathematical physics, representation theory, etc. Although at first sight their definition might look arkward, they appear naturally as derivation algebras of any kind of non-necessarily associative algebras. Many results on Lie algebras can be proved by elementary methods like linear algebra and euclidean geometry. Nevertheless, many results on Lie algebras have a strong impact in other mathematical areas as well.

The central subject of the course will be the classification of finite-dimensional semi-simple Lie algebra over algebraically closed fields of characteristic 0 which is essentially due to E. Cartan. Apart from several important facts on Lie algebras of this type, the classification is achieved by a detailed
analysis of the easiest example sl_2 of such a Lie algebra which allows to transform the classification problem to the classification of finite euclidean reflection groups. The latter problem can be resolved by euclidean geometry and combinatorics.

The course will be divided in two parts which will be developed independently, and simultaneously: The structure theory of semi-simple Lie algebras, and the classification of finite, chrystallographic root systems. Indeed, many problems on semi-simple Lie algebras can be transferred to an equivalent problem in terms of these root systems.

It should be mentioned that although one should consider Cartan's classification theorem as a classical result, it has paved the way to many new mathematical areas which are today the subject of intensive research activity.

The course will be given in English.

R. W. Carter, Lie Algebras of Finite and Affine Type, Cambridge Studies in Advanced Mathematics, 96, Cambridge University Press, Cambridge, 2005.

J. E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Texts in Mathematics, 9, Springer-Verlag, New York, 1978.

J. E. Humphreys, Reflection Groups and Coxeter Groups, Cambridge Studies in Advanced Mathematics, 27, Cambridge University Press, Cambridge, 1990.

N. Jacobson, Lie Algebras, Dover Publications, Inc., New York, 1979.