AN INTRODUCTION TO CALABI'S EXTREMAL METRICS

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- **Elements of Kähler geometry:** Basic definitions, Kähler identities, dd^c-lemma, Ricci form, holomorphic vector fields, Lichnerowicz fourth-order operator etc.
- **Calabi extremal metrics:** definitions and main properties.
- **Geometry of the space of Kähler metrics:** Mabuchi energy, Futaki invariant, geodesics, Donaldson's generalized Futaki invariahnt, etc...
- **Examples of extremal metrics:** constructions and links with stability conditions.

References:

(1) E. Calabi, *On Kähler manifolds with vanishing canonical class*, in *Algebraic Geometry and Topology : A symposium in honour of Lefschetz*, Princeton University Press (1955), 78–89.

(2) E. Calabi, *Extremal Kähler metrics*, in *Seminar of Differerential Geometry*, ed. S. T. Yau, Annals of Mathematics Studies **102**, Princeton University Press (1982), 259–290.

(3) Paul Gauduchon, *Calabi's extremal metrics: An elementary introduction* (work in progress).

(4) Andrei Moroianu, *Lectures on Kähler geometry.* London Mathematical society Student Texts **69**, Cambridge University press, cambridge (2007).

(5) André Weil, *Introduction à l'étude des variétés kälériennes*, Actualités Scientifiques et Industrielles *1267*, Publications de l'Université de Nancago, VI, Hermann Paris 91958).