

*Scuola Matematica Interuniversitaria*

*Cortona, Italy*

*July-August 2014*

**Qualitative properties and geometric aspects  
of solutions to nonlinear PDEs**

*Alberto FARINA*

Université de Picardie Jules Verne

LAMFA, CNRS UMR 7352

Amiens, France

**SYLLABUS**

- Motivations and examples.
- Monotonicity and one-dimensional symmetry on the entire euclidean space and in half-spaces: Gibbons' conjecture and some open questions.
- Pointwise gradient estimates in general domains and consequences : Liouville-type theorems, classification of the domains and symmetry of the solutions.
- A geometric Poincaré-type formula and its consequences :
  - i) Overdetermined boundary value problems on unbounded domains and a conjecture of Berestycki, Caffarelli and Nirenberg,
  - ii) a conjecture of De Giorgi in dimension 2 and 3 (for a general nonlinearity).
- Extensions to systems and manifolds.

**BACKGROUND**

Basic knowledge of : Partial Differential Equations, Real and Functional Analysis.

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